Alfateh University

Electrical Engineering Department

EE303 Numerical Analysis

Mid-Term I

Answer all questions, Carry calculations to 3 decimal places, time allowed 1.5 hours

Q(1)

a) Given $\sin(x)=x-(1/3!)x^3+(1/5!)x^5-(1/7!)x^7+\cdots$ Write a recursive expression in the form of $T_{i+1}=(\ldots)T_i$ (3Marks)

b) Write a *C program* to find the root of a nonlinear equation using the secant method. (4Marks)

c) Starting at [7,16], how many iterations are needed for the absolute error to drop down to 10^{-9} using the Bisection method (3Marks)

(Q2)

a) Use Newton's method to find the root of the following function

$f(x)=\tan(x)-30x$

Start with $x_0=1$ and perform only three iterations (5 Marks)

b) Using Taylor's theorem, show that the error at the $n+1$ iteration can be approximated by: $e_{i+1} \approx \frac{-f''(r)}{2f'(r)}e_i^2$ (5 Marks)

(Q3) Let $A=\begin{bmatrix}-1 & 4 & -4\\ 2 & 2 & 0\\ 3 & 3 & 2\end{bmatrix}$, $x=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$ and $b=\begin{bmatrix}0\\ 1\\ 1\\ 2\end{bmatrix}$

a) Solve the system using Gaussian elimination without pivoting and find the L matrix from the steps of Gaussian elimination (5 Marks)

b) If A can be factorized as $A=LU$, solve the system $Ax=b$ using the following two stages (5 Marks)

$Lz=b$ solve for z

$Ux=z$ solve for x

Spring 2007

Dr. Idris El-Feghi

Mid-Term (I) Spring 2007

Q1:- Given $\sin(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7$

$T_{i+1} = (\quad) T_i$ $i = 1$

$T_2 = (\quad) T_1$ i_2

$\frac{x^5}{5!} = \left(\frac{(-1)^i x^2}{(2i+3)(2i+2)}\right) \frac{-x^3}{3!}$ $(-1)^i$ $(-1)^1 = -1$

$3 \times \boxed{3!}$

$n+3$ $n+2$ $\times$

3×2

$+2$

3

$\frac{x^2}{(2+3)(2+2)}$ $\frac{x^3}{6}$

$i = 2$

$T_{i+1} = \left(\frac{(-1)^2 x^2}{(2\times2+3)(2\times2+2)}\right) T_2$

(0, 2, 4) 3×2

ز ف

x^5

1 و 3

5×4

$\frac{x^7}{7!} = \left(\frac{x^2}{7 \times 6}\right) \frac{x^5}{5!}$

يساوي صفر

cos 90

$\frac{x^7}{7!}$

1:- C

[7, 16] $n = ??$ $\boxed{17}$

$(b_{n+1} - a_{n+1}) = 10^{-9}$

$10^{-9} = 2^{-n}(16-7)$

$\log \frac{10^{-9}}{9} = \log \frac{2^{-n}(9)}{9}$

$\log \frac{10^{-9}}{9} = \log 2^{-n}$

$\log 0{,}11111 = -n \log 2$

$n = \frac{-\log 0{,}1111 \times 10^{-10}}{-\log 2} =$

$n =$

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$f(x) = \tan(x) - 30x$ start $x_0 = 1$

3 iteration

$$f'(x_n) = \frac{f(x_n)}{x_n - x_{n+1}}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\boxed{f'(x) = \sec^2 x - 30}$$

$$\sec^2 x = \frac{1}{\cos^2 x}$$

$$= \frac{1}{\cos^2(1)} - 30$$

$\tan(1) - 30(1)$

x	$f(x)$	$f'(x)$
1	-28.44	-29.45
0.344	-0.9947	-29.45

ثم نعوض في القانون

لكي نحصل على x الجديد

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= 1 - \frac{-28.44}{-29.45}$$

وهذا لكي نحصل على باقي القيم x الجديدة

ثم نعوض بها في $f(x)$

ونعوض بها في $f'(x)$

واشتقتها وهكذا

$\boxed{20}$

Let $A = \begin{bmatrix} -1 & 1 & -4 \\ 2 & 2 & 0 \\ 3 & 3 & 2 \end{bmatrix}$ $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ 1 \\ \frac{1}{2} \end{bmatrix}$

[illegible]

$$\begin{array}{cccc} -2 & 2 & -8 & 0 \end{array}$$
$$\left[\begin{array}{ccc|c} -1 & 1 & -4 & 0 \\ 2 & 2 & 0 & 1 \\ 3 & 3 & 2 & \frac{1}{2} \end{array}\right] = R_2 = 2R_1 + R_2 \left[\begin{array}{ccc|c} -1 & 1 & -4 & 0 \\ 0 & 4 & -8 & 1 \\ 3 & 3 & 2 & \frac{1}{2} \end{array}\right]$$

$$\begin{array}{cccc} -3 & 3 & -12 & 0 \end{array}$$
$$R_3 = 3R_1 + R_3 = \left[\begin{array}{ccc|c} -1 & 1 & -4 & 0 \\ 0 & 4 & -8 & 1 \\ 0 & 6 & -10 & 0.5 \end{array}\right]$$

$$= 6R_2 - 4R_3 = \left[\begin{array}{ccc|c} -1 & 1 & -4 & 0 \\ 0 & 4 & -8 & 1 \\ 0 & 0 & 68 & -4 \end{array}\right]$$

$$\begin{bmatrix} -1 & 1 & -4 \\ 0 & 4 & -8 \\ 0 & 0 & 68 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix}$$

$$68x_3 = -4 \Rightarrow x_3 = -\frac{4}{68} = 0.05$$

$$4x_2 + 8x_3 = 1$$
$$4x_2 - 8(0.05) = 1$$
$$x_2 = \frac{1+}{4} \Rightarrow x_2 = 0.3 \quad -0.75$$

$$-x_1 + x_2 - 4x_3 = 0 \qquad x_1 = 1.25$$

3 b

if a can be factorized as $A = lu$ solve the system $Ax = b$ using the following two stages

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$lz = h$ solve for z $\quad A = lu$

$$A = \begin{bmatrix} -1 & 1 & -4 \\ 2 & 2 & 0 \\ 3 & 3 & 2 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 1 \\ 0.5 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & -4 \\ 2 & 2 & 0 \\ 3 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$\boxed{u_{11} = -1}$ $\boxed{u_{12} = 1}$ $\boxed{u_{13} = -4}$

$l_{21} u_{11} = 2$

$-l_{21} = 2$

$\boxed{l_{21} = -2}$

$l_{21} u_{12} + u_{22} + 0 = 2$

$(-2)(1) + u_{22} = 2$

$u_{22} = 2 + 2 = 4$

$\boxed{u_{22} = 4}$

$u_{11} l_{31} = 3$

$-l_{31} = 3$

$\boxed{l_{31} = -3}$

$u_{12} l_{31} + l_{32} u_{22} = 3$

$-3 + 4 l_{32} = 3$

$4 l_{32} = 6$

$\boxed{l_{32} = \frac{6}{4} = \frac{3}{2}}$

$l_{21} u_{13} + u_{23} = 0$

$-2(-4) + u_{23} = 0$

$8 + u_{23}$

$\boxed{u_{23} = -8} \Rightarrow$

$\cancel{12} = 12 + \boxed{u_{33} = 2}$

$$L = \begin{bmatrix} & 0 & 0 \\ 2 & 1 & 0 \\ 3 & \frac{3}{2} & 1 \end{bmatrix} \qquad U = \begin{bmatrix} -1 & 1 & -4 \\ 0 & 4 & -8 \\ 0 & 0 & 2 \end{bmatrix}$$

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$AX = b \qquad \therefore A = LU$

$L(UX) = b \qquad UX = Z$

$L^{-1}L(UX) = L^{-1}b \qquad b = LZ$

$\therefore Z = L^{-1}b$

$LL^{-1} =$ مصفوفة الوحدة

$$\begin{bmatrix} & 0 & 0 \\ -2 & 1 & 0 \\ 3 & \frac{3}{2} & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\boxed{a_{11} = 1} \quad \boxed{a_{12} = 0} \quad \boxed{a_{13} = 0}$

$a_{11} + a_{21} = 0$

$-2 + a_{21} = 0$

$\boxed{a_{21} = 2}$

$-2a_{12} + a_{22} + 0 = 1$

$0 + a_{22} = 1 \quad \boxed{a_{22} = 1}$

$a_{11} + \frac{3}{2}a_{21} + a_{31} = 0$

$+(\frac{3}{2})(2) + a_{31} = 0$

$\boxed{a_{31} = 0}$

$-3a_{12} + \frac{3}{2}a_{22} + a_{32} = 0$

$-3(0) + \frac{3}{2}(1) + a_{32} = 0$

$\frac{3}{2} + a_{32} = 0 \quad \therefore \boxed{a_{32} = -\frac{3}{2}}$

$-2a_{13} + a_{23} = 0$

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so $L^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & \frac{-3}{2} & 1 \end{bmatrix}$

$$\overset{L^{-1}}{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -\frac{3}{2} & 1 \end{bmatrix}} \overset{b}{\begin{bmatrix} 0 \\ 1 \\ \frac{1}{2} \end{bmatrix}} \overset{=}{=} \overset{Z}{\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}}$$

$$UX = Z$$

$$\begin{array}{l} -\frac{3}{2}+\frac{1}{2} \\ -\frac{2}{2} \end{array} \begin{bmatrix} -1 & 1 & -4 \\ 0 & 4 & -8 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$-X_1 + X_2 - 4X_3 = 0$

$4X_2 - 8X_3 = 1$

$2X_3 = -1$

$\boxed{\text{so } X_3 = -\frac{1}{2}}$

$4X_2 + \cancel{8}^{4} X \frac{1}{\cancel{2}} = 1$

$4X_2 = 1 - 4$

$-X_1 \cancel{+} 0.75 + 2 = 0$

$-X_1 = +0.75 - 2$

$\boxed{X_1 = +1.25}$